Optimal pumping of orbital entanglement with single particle emitters

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We propose a method for the optimal time-controlled generation of entangled itinerant particles, using on-demand sources in a conductor in the quantum Hall regime. This entanglement pump is realized by applying periodic, tailored voltage pulses to pairs of quantum dots or quantum point contacts. We show that the pump can produce orbital Bell pairs of both electrons and holes at the optimal rate of half a pair per pumping cycle. The entanglement can be detected by a violation of a Bell inequality formulated in terms of low-frequency current cross correlations.

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Entanglement of itinerant electrons in mesoscopic and nanoscale systems continues to attract great attention. Despite intense effort, a clear demonstration of the generation, spatial separation and detection of entangled pairs of electrons is still missing. Of particular interest would be an on-demand source for entanglement [1–4], while a key component in quantum information processing is the time-controlled production of entangled flying quantum bits. A scheme for the dynamical generation, or pumping, of orbitally entangled electron-hole pairs was proposed by two of us in [1]. However, as the proposed pump operated in the weak amplitude regime, it produced on average much less than one Bell pair per cycle. Subsequently it was shown that, for strong amplitude pumping of non-interacting particles, the optimal production rate is half a Bell pair per cycle [2]. Optimal electron-hole pair entanglement pumps have also been proposed, but without any scheme for entanglement detection [2, 4].

An important step towards entanglement pumping was taken experimentally by Fève *et al* [5], who realized a single-particle on-demand source in a conductor in the quantum Hall regime [6–11]. Under ideal conditions, the source produces exactly one electron and one hole per cycle. A scheme for the production of pairs of orbitally entangled particles in different time bins based on two on-demand sources in a double electronic Mach-Zehnder interferometer was proposed in [12, 13]. The origin of the entanglement was two-particle interference [14–16], manifested by a non-local two-particle Aharonov-Bohm effect. The maximum production was 1/4 Bell pair per cycle, *i.e.* half the optimal rate.

Here we propose an entanglement pump, aiming for the simplest scheme for detection and optimal production of orbital entanglement in the same system, see Fig. 1. A conductor in the quantum Hall regime with two on-demand sources, C and D, is connected to four terminals via electronic beam-splitters at A and B. Using a single spin-polarized edge state and the interferometer of [1], the sources operate in the strong amplitude regime and generate pairs of entangled particles at an optimal rate.

The two types of sources, driven by voltage pulses

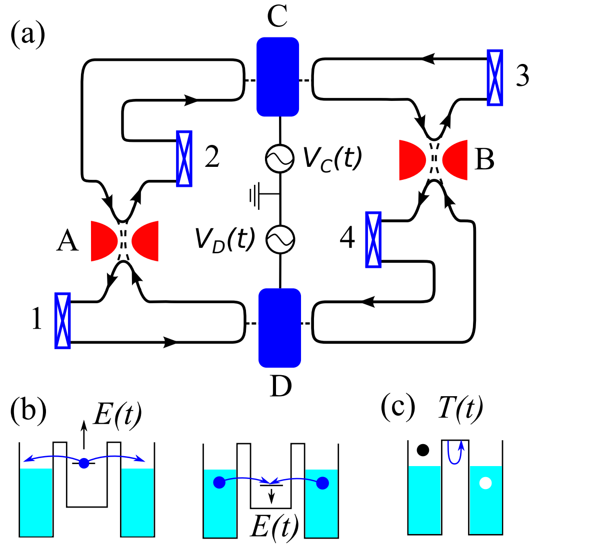

Figure 1: a) Optimal orbital entanglement pump with two single particle emitters, C and D, connected to conductors in the quantum Hall regime. The emitted electrons/holes propagate along edge states to controllable electronic beam splitters at A and B and are detected at terminals 1-4. The sources C and D can be either b) quantum dots or c) quantum point contacts. Tailored voltage profiles $V_C(t)$ and $V_D(t)$ are applied to a gate in case b) to move a localized level with energy $E(t)$ up (down) through the Fermi energy releasing an electron (hole), or in case c) to cycle the transmission probability, $T(t)$, from zero through unity and back to generate a particle-hole excitation.

$V_C(t)$ and $V_D(t)$, are shown in Figs. 1(b) and (c). Fig. 1(b) shows the level of a quantum dot (QD) driven up (down) through Fermi energy, generating a single electron (hole), as in the experiment of [5]. However, with the QD coupled to separate quantum Hall edge states, the particle is emitted into a superposition of states at the two edges. In Fig. 1(c), a quantum point contact (QPC) is opened and closed to generate exactly one particle and one hole, which are independently emitted into edge state superpositions. We show below that, for QD

as well as QPC sources, driving C and D periodically generates orbitally entangled pairs of both electron and hole wavepackets, with one particle propagating towards A and one towards B. For synchronized and spatially symmetric sources the entanglement production rate is optimal, with half a Bell pair per cycle. Using earlier results for many-body states of emitted particles of on-demand sources [4, 6, 9], we derive explicit expressions for the entangled wavefunction. The entanglement, arising from two-particle interference [14–16], can be detected via low-frequency cross correlations of currents at the four terminals 1-4. Throughout the paper consider zero temperature and put $\hbar, e = 1$.

QD-sources: We first consider QD-sources. The aim is to obtain the many-body wavefunction of the entangled particles emitted from the sources towards A and B. We consider the scattering properties of source C for a single electron emission event. The QD has a single localized level at energy $E(t)$ and tunnel couplings to the edges towards A and B. If the energy of the level is varied slowly on the scale of the Wigner delay time (adiabatic approximation), the time-development of the scattering states is given by the instantaneous value of the scattering matrix, S_C. When the energy of the level is varied at constant speed, $E(t) = \nu(t - t_C)$, S_C takes the Breit-Wigner form:

$$S_C = \frac{1}{t - t_C - i\tau_C} \begin{pmatrix} t - t_C + i\bar{\tau}_C & -2i\sqrt{\tau_{CA}\tau_{CB}} \\ -2i\sqrt{\tau_{CA}\tau_{CB}} & t - t_C - i\bar{\tau}_C \end{pmatrix} \quad (1)$$

with $\tau_C = \tau_{CA} + \tau_{CB}$ and $\bar{\tau}_C = \tau_{CA} - \tau_{CB}$. Here $\nu\tau_{CA}$ and $\nu\tau_{CB}$ are tunnel rates through the barriers into states propagating towards A and B.

The scattering matrix, S_C, can be diagonalized in a time-independent basis, $\tilde{S}_C = V S_C V^\dagger = \begin{pmatrix} e^{i\phi_C} & 0 \\ 0 & 1 \end{pmatrix}$, with $V = \frac{1}{\sqrt{\tau_C}} \begin{pmatrix} -\sqrt{\tau_{CA}} & \sqrt{\tau_{CB}} \\ \sqrt{\tau_{CB}} & \sqrt{\tau_{CA}} \end{pmatrix}$ and

$$e^{i\phi_C(t)} = \frac{t - t_C + i\tau_C}{t - t_C - i\tau_C}. \quad (2)$$

The low temperature many-body state incident on C is a filled Fermi sea of electrons originating from terminals 2 and 3. The incident many-body state is $|\psi_{in}\rangle = \prod_{\epsilon<0} a^\dagger_{C+}(\epsilon) a^\dagger_{C-}(\epsilon)|\rangle$, where the $a^\dagger_{C\pm}(\epsilon)$ are creation operators in the diagonal basis for particles with energy ϵ in the incoming channels $+$ or $-$, and $|\rangle$ is the vacuum. Denoting the operators for particles in the corresponding outgoing states by $b^\dagger_{C\pm}(\epsilon)$, the many-body state after impinging on C is

$$|\psi_C\rangle = \prod_{\epsilon<0} b^\dagger_{C+}(\epsilon) \sum_{\epsilon'} (e^{i\phi_C})_{\epsilon,\epsilon'} b^\dagger_{C-}(\epsilon')|\rangle. \quad (3)$$

Here $(e^{i\phi_C})_{\epsilon,\epsilon'}$ is the Fourier transform of $e^{i\phi_C(t)}$ with respect to the energy difference $(\epsilon - \epsilon')$ [17–19].

With the phase profile of (2) and taking $\nu > 0$, the many-body state becomes $|\psi^e_C\rangle = \sqrt{2\tau_C} \sum_{\epsilon>0} \exp[\epsilon(it_C - \tau_C)] b^\dagger_{C-}(\epsilon)|0\rangle$. It describes a single electron above the unperturbed Fermi sea $|0\rangle$ [6, 9]. The wavefunction of the single particle excitation is $\psi(x,t) \sim 1/(x - v_{dr}[t - t_C - i\tau_C])$, where x is the distance from C and v_{dr} the drift velocity along the edge. Using $(b^\dagger_{CA}(\epsilon), b^\dagger_{CB}(\epsilon)) = V(b^\dagger_{C-}(\epsilon), b^\dagger_{C+}(\epsilon))$ to write $|\psi^e_C\rangle$ in terms of operators $b^\dagger_{CA}(\epsilon)$ and $b^\dagger_{CB}(\epsilon)$, which create particles propagating along the edges towards A and B, we obtain

$$|\psi^e_C\rangle = \frac{1}{\sqrt{\tau_C}} \left[-\sqrt{\tau_{CA}} p^\dagger_{CA} + \sqrt{\tau_{CB}} p^\dagger_{CB} \right] |0\rangle. \quad (4)$$

Here the normalized wavepacket operator $p^\dagger_{CA} = \sqrt{2\tau_C} \sum_{\epsilon>0} \exp[\epsilon(it_C - \tau_C)] b^\dagger_{CA}(\epsilon)$ and similarly for $p^\dagger_{CB}$. Eq (4) shows that driving the QD-level at C up through the Fermi level injects a single electron into a linear superposition of states propagating towards A and B.

The corresponding result to (4), when driving the QD level at D up through the Fermi level, is $|\psi^e_D\rangle = (1/\sqrt{\tau_D}) \left[-\sqrt{\tau_{DA}} e^{-i\eta} p^\dagger_{DA} + \sqrt{\tau_{DB}} e^{i\eta} p^\dagger_{DB} \right] |0\rangle$. Here $p^\dagger_{DA}$ is obtained from $p^\dagger_{CA}, p^\dagger_{DB}$ by changing indices $C \leftrightarrow D$. We have also included a phase-factor $e^{i\eta}$ to account for a possible Aharonov Bohm phase. The total wave function for the electrons emitted from C and D is then the product $|\psi^{ee}_{out}\rangle = |\psi^e_C\rangle|\psi^e_D\rangle$. We are interested in the non-local properties of $|\psi^{ee}_{out}\rangle$, with the particles emitted towards different beamsplitters. Projecting $|\psi^{ee}_{out}\rangle$ onto the subspace with one particle at A and one at B, gives the normalized wavefunction [20]

$$\begin{aligned} |\psi^{ee}_{AB}\rangle &= \frac{1}{\sqrt{N}} \Big[\sqrt{\tau_{CA}\tau_{DB}} p^\dagger_{CA} p^\dagger_{DB} e^{i\eta} \\ &+ \sqrt{\tau_{CB}\tau_{DA}} p^\dagger_{CB} p^\dagger_{DA} e^{-i\eta} \Big] |0\rangle \end{aligned} \quad (5)$$

where $N = \tau_{CA}\tau_{DB} + \tau_{CB}\tau_{DA}$. The weight, or probability that $|\psi^{ee}_{AB}\rangle$ is generated, is $w_{AB} = N/(\tau_C\tau_D)$.

The wavefunction $|\psi^{ee}_{AB}\rangle$ describes two particles above the filled Fermi sea. It is entangled in the orbital (*i.e.* source C and D) degree of freedom—a result of two-particle interference [14]. The superposition in $|\psi^{ee}_{AB}\rangle$ results from the indistinguishability of the two emission processes leading to a particle at A and one at B. In one process a particle moves from C to A and another from D to B. In the second process particles move from D to A and from C to B. To quantify the orbital entanglement we consider the reduced 4×4 orbital density matrix ρ_{AB}. This is obtained by tracing $|\psi^{ee}_{AB}\rangle\langle\psi^{ee}_{AB}|$ over energy [21]:

$$\begin{aligned} \rho_{AB} &= N^{-1} \big(\tau_{CA}\tau_{DB} |CD\rangle\langle CD| + \tau_{CB}\tau_{DA} |DC\rangle\langle DC| \\ &+ \chi\sqrt{\tau_{CA}\tau_{CB}\tau_{DA}\tau_{DB}} \left[e^{i\eta} |CD\rangle\langle DC| + h.c. \right] \big) \end{aligned} \quad (6)$$

where $|CD\rangle \equiv |C\rangle_A |D\rangle_B$ and $|C\rangle_A$ denotes an electron at A emitted from C etc. Here

$$\chi = \frac{4\tau_C\tau_D}{(t_C - t_D)^2 + (\tau_C + \tau_D)^2} \quad (7)$$

quantifies the overlap ($0 \leq \chi \leq 1$) between the two wavepackets emitted from C and D. We note that a reduced overlap $\chi < 1$ plays the same role as a non-zero dephasing for two-particle interference [16].

The entanglement of ρ_{AB} is conveniently quantified via the concurrence $\mathcal{C}$, which ranges from 0, for a separable non-entangled state, to 1 for a maximally entangled state [22] . For ρ_{AB} we find

$$\mathcal{C} = 2(\chi/N)\sqrt{\tau_{CA}\tau_{CB}\tau_{DA}\tau_{DB}} \tag{8}$$

which is non-zero for arbitrary small overlap χ and arbitrary tunnel couplings [12]. For QDs synchronized in time, $t_C = t_D$, with symmetric couplings $\tau_{CA} = \tau_{CB} = \tau_C/2$ and $\tau_{DA} = \tau_{DB} = \tau_D/2$, the density matrix $\rho_{AB} = |\varphi^{ee}_{AB}\rangle\langle\varphi^{ee}_{AB}|$ with $|\varphi^{ee}_{AB}\rangle = 1/\sqrt{2}[e^{i\eta}|CD\rangle + e^{-i\eta}|DC\rangle]$. This state is an orbital Bell pair, *i.e.* it is maximally entangled ($\mathcal{C} = 1$). Moreover, the weight $w_{AB} = 1/2$ and hence the concurrence production per cycle is $w_{AB}\mathcal{C} = 1/2$, the theoretical maximum [20].

If the QD-level at C is driven down through the Fermi level at time t'_C, a hole is generated in a linear superposition of states in the edges towards A and B, with a wavefunction $|\psi^h_C\rangle = (1/\sqrt{\tau_C})\left[-\sqrt{\tau_{CA}}h^\dagger_{CA} + \sqrt{\tau_{CB}}h^\dagger_{CB}\right]|0\rangle$. Here the hole wavepacket creation operator is $h^\dagger_{CA} = \sqrt{2\tau_C}\sum_{\epsilon<0}\exp[\epsilon(it'_C + \tau_C)]b_{CA}(\epsilon)$. A similar relation holds for $|\psi^h_D\rangle$, giving a total hole wavefunction $|\psi^{hh}_{out}\rangle = |\psi^h_C\rangle|\psi^h_D\rangle$.

For a large amplitude driving up and subsequently down through the Fermi energy, with $t'_C - t_C \gg \tau_C$ and $t'_D - t_D \gg \tau_D$, the electron and hole emissions are well separated in time and can be treated as independent [8, 9]. The total wavefunction for the emitted particles is then $|\psi^{QD}_{out}\rangle = |\psi^{ee}_{out}\rangle|\psi^{hh}_{out}\rangle$. In the experimentally relevant situation [5], with a cycling of the QD-level with period $T \gg \tau_C, \tau_D$, the pump produces pairs of both entangled electrons and holes which, for symmetric and syncronized sources, reaches the optimal production rate of 1/2 a Bell pair per period.

QPC-sources. For QPC sources, the emitted state is created by applying tailored voltage pulses $V_{C,D}(t)$ to the gates C and D [4]. The instantaneous scattering matrix of gate C can be written

$$S_C = \begin{pmatrix} \lambda_C(t) & \kappa_C(t) \\ -\kappa^*_C(t) & \lambda^*_C(t) \end{pmatrix} \tag{9}$$

where $|\kappa_C(t)|^2$ and $|\lambda_C(t)|^2$ are the transmission and reflection probabilities through the QPC respectively. Provided no voltage is applied across the device, S_C can be diagonalized [4] in a time-independent basis $\tilde{S}_C = VS_CV^\dagger = \begin{pmatrix} e^{i\phi_C} & 0 \\ 0 & e^{-i\phi_C} \end{pmatrix}$, with $V = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix}$ and $e^{i\phi_C(t)} = \lambda_C(t) + i\kappa_C(t)$. The outgoing manybody wavefunction can then be written in the diagonal basis

$$|\bar{\psi}_C\rangle = \frac{1}{2}\prod_{\epsilon<0}\sum_{\epsilon',\epsilon''}(e^{i\phi_C})_{\epsilon,\epsilon'}(e^{-i\phi_C})_{\epsilon,\epsilon''}b^\dagger_-(\epsilon')b^\dagger_+(\epsilon'')|\rangle. \tag{10}$$

By varying $V_C(t)$ so that the transmission amplitude $\kappa_C(t) = 2\tau_C(t - t_C)/[(t - t_C)^2 + \tau_C^2]$, the phase factor $e^{i\phi_C(t)}$, is again given by (2). In this case we can write the outgoing manybody wavefunction [6]

$$|\psi^{eh}_C\rangle = \frac{1}{2}\left(p^\dagger_{CA} + ip^\dagger_{CB}\right)\left(h^\dagger_{CA} - ih^\dagger_{CB}\right)|0\rangle \tag{11}$$

in the unrotated basis with a corresponding result for $|\psi^{eh}_D\rangle$. The wavefunctions $|\psi^{eh}_{C,D}\rangle$ describes one electron and one hole wavepacket, independently emitted on top of the unperturbed Fermi sea, in superpositions which describe excitations propagating towards A and B. Consequently, the total state $|\psi^{QPC}_{out}\rangle = |\psi^{eh}_C\rangle|\psi^{eh}_D\rangle$, as for the QD, is the direct product of independently emitted pairs of electrons and holes. The difference is that for the QPC the electrons and holes at C (D) are emitted at the same time, $t'_C = t_C$ and $t'_D = t_D$. The reduced density matrix and the concurrence of the electron and hole states at A and B are given by Eqs. (6) and (8) respectively, after taking $\tau_{CA} = \tau_{CB} = \tau_C/2$ and $\tau_{DA} = \tau_{DB} = \tau_D/2$. For perfectly synchronized driving $t_C = t_D$ the QPC also works as an optimal entanglement pump, producing independent pairs of entangled electrons and holes at A and B with a rate 1/2 per cycle.

Bell inequality The existence of entanglement in the system illustrated in Fig. 1 can be verified experimentally by demonstrating violation of a Bell inequality. A BI can conveniently be formulated in terms of the joint, or coincident, probabilities to detect quasiparticle excitations [14], as $M \leq 2$ for the Bell parameter

$$M = |E(\alpha,\beta) - E(\alpha,\beta') + E(\alpha',\beta) + E(\alpha',\beta')|. \tag{12}$$

Here the correlation function

$$E(\alpha,\beta) = \frac{P_{24} + P_{13} - P_{14} - P_{23}}{P_{24} + P_{13} + P_{14} + P_{23}}, \tag{13}$$

where P_{ij} ($= P^{ee}_{ij} + P^{hh}_{ij}$) is the sum of the probabilities to jointly detect an electron/hole at A ($i = 1, 2$) and B ($i = 3, 4$) during a pumping cycle [1], and is given by

$$\begin{aligned} P^{ee}_{ij} &= \int_0^\infty d\epsilon\int_0^\infty d\epsilon'\langle b^\dagger_i(\epsilon)b^\dagger_j(\epsilon')b_j(\epsilon')b_i(\epsilon)\rangle, \\ P^{hh}_{ij} &= \int_{-\infty}^0 d\epsilon\int_{-\infty}^0 d\epsilon'\langle b_i(\epsilon)b_j(\epsilon')b^\dagger_j(\epsilon')b^\dagger_i(\epsilon)\rangle. \end{aligned} \tag{14}$$

The annihilation operators for the states entering the terminals i, j at A are

$$\begin{pmatrix} b_2 \\ b_1 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} b_{CA} \\ b_{DA} \end{pmatrix}. \tag{15}$$

The operators at B are obtained after setting $\alpha \to \beta$, $1 \to 3$ and $2 \to 4$. From the emitted manybody wavefuctions for the QD and the QPC, together with the operator relations at the beamsplitters (15), we obtain the joint detection probability

$$P_{13}^{ee} = (2\tau_{CA}\tau_{DB}\sin^2\alpha\cos^2\beta + 2\tau_{CB}\tau_{DA}\sin^2\beta\cos^2\alpha - \chi\cos\eta\sqrt{\tau_{CA}\tau_{DB}\tau_{CB}\tau_{DA}}\sin 2\alpha\sin 2\beta)/(2\tau_C\tau_D) \quad (16)$$

with corresponding results for the other P_{ij}^{ee} ($P_{ij}^{hh} = P_{ij}^{ee}$) [23]. For the QPC-source we put $\tau_{CA} = \tau_{CB} = \tau_C/2$ and $\tau_{DA} = \tau_{DB} = \tau_D/2$. Inserting the P_{ij} into (13) gives

$$E(\alpha, \beta) = -(\cos 2\alpha\cos 2\beta + \mathcal{C}\cos\eta\sin 2\alpha\sin 2\beta) \quad (17)$$

where $\mathcal{C}$ is the concurrence in (8). By optimising the settings α, α', β and β' of the beamsplitters A and B [24], the BI reduces to $2\sqrt{1+\mathcal{C}^2\cos^2\eta} \leq 2$ which can be violated for arbitrary overlap χ and phase η.

The joint detection probabilities in (14) are presently not directly accessible in mesoscopic conductors, as they require time-resolved correlation measurements on the time scale of the period T. It is however possible to express P_{ij} in terms of experimentally available currents and low frequency current cross-correlators. The period-averaged, zero frequency cross-correlations S_{ij} [25] are found to be

$$S_{ij} = T^{-1}\left(P_{ij}^{ee} - P_i^e P_j^e + P_{ij}^{hh} - P_i^h P_j^h\right). \quad (18)$$

Note that S_{ij} does not contain any electron-hole correlations, as a consequence of the independent emission of electrons and holes, discussed above. In (18) the (single particle) probability to detect an electron in lead i is $P_i^e = \int_0^\infty d\epsilon \langle b_i^\dagger(\epsilon) b_i(\epsilon)\rangle$, while P_i^h is the probability to detect a hole. For the QD P_i^e and P_i^h are available from $I_i(t)$, which is the experimentally accessible time-resolved current [5]. By integrating over half-cycles, driving the QD-level up (down) during the first (second) half, we have $(1/e)\int_0^{T/2} dt I_i(t) = Q_i^e/e = P_i^e$ and $(1/e)\int_{T/2}^{T} dt I_i(t) = Q_i^h/e = P_i^h$. Here Q_i^e and Q_i^h are the electron and hole charge flowing into contact i during the cycle. For the QPC P_i^e and P_i^h can instead be obtained in a more indirect way via the low-frequency current autocorrelations, which we do not discuss here. Taken together, this allows us to express the correlation function $E(\alpha, \beta)$, and hence the Bell inequality, directly in terms of currents and current correlations.

In conclusion, we have proposed an optimal entanglement on-demand source in a non-interacting mesoscopic conductor in the quantum Hall regime. Pairs of entangled, spatially separated electrons and holes are generated by applying tailored, time-dependent voltage pulses to quantum dots or quantum point contacts. The entanglement can be investgated by current and current cross correlation measurements.

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